

GUJARAT TECHNOLOGICAL UNIVERSITY**DIPLOMA ENGINEERING – SEMESTER – 2(NEW) • EXAMINATION – SUMMER - 2018****Subject Code: 3320002****Date: 23-May-2018****Subject Name: ADVANCED MATHEMATICS (GROUP-1)****Time: 10:30 AM TO 01:00 PM****Total Marks: 70****Instructions:**

1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible. (Scientific/Higher Version not allowed)
5. English version is authentic.

Q.1

Fill in the blanks using appropriate choice from the given options.

141 If $|\bar{z}| = 16$, then $|z| = \dots\dots\dots$

- (a) 16 (b) 4 (c) 256 (d) 1

૧ જો $|\bar{z}| = 16$ હોય, તો $|z| = \dots\dots\dots$

- (a) 16 (b) 4 (c) 256 (d) 1

2 $i^9 = \dots\dots\dots$

- (a) i (b) -i (c) 1 (d) -1

૨ $i^9 = \dots\dots\dots$

- (અ) i (બ) -i (ક) 1 (ડ) -1

3 If $z = 5 - 2i$ then $\bar{z} = \dots\dots\dots$

- (a)
- $-5 + 2i$
- (b)
- $-5 - 2i$
- (c)
- $5 + 2i$
- (d) none of these

૩ જો $z = 5 - 2i$ હોય તો $\bar{z} = \dots\dots\dots$

- (અ)
- $-5 + 2i$
- (બ)
- $-5 - 2i$
- (ક)
- $5 + 2i$
- (ડ) એક પણ નહીં

4 If $f(x) = \log(\tan x)$ then $f\left(\frac{\pi}{4}\right) = \dots\dots\dots$

- (a) 1 (b) e (c) 0 (d)
- π

૪ જો $f(x) = \log(\tan x)$ તો $f\left(\frac{\pi}{4}\right) = \dots\dots\dots$

- (a) 1 (b) e (c) 0 (d)
- π

5 $\lim_{\theta \rightarrow 0} \frac{\theta}{\tan 3\theta} = \dots\dots\dots$

- (a) 3 (b)
- $\frac{1}{3}$
- (c) 1 (d) 0

૫ $\lim_{\theta \rightarrow 0} \frac{\theta}{\tan 3\theta} = \dots\dots\dots$

- (a) 3 (b)
- $\frac{1}{3}$
- (c) 1 (d) 0

6 $\frac{d}{dx}(x^2+2^x+2^2) = \dots\dots\dots$
 (a) 1 (b) $2x+2^x+2^2$ (c) $2x+2^x\log 2$ (d) 0

5 $\frac{d}{dx}(x^2+2^x+2^2) = \dots\dots\dots$
 (a) 1 (b) $2x+2^x+2^2$ (c) $2x+2^x\log 2$ (d) 0

7 If $y = e^x$, then $\frac{d^2y}{dx^2} = \dots\dots\dots$
 (a) e^x (b) e^{2x} (c) e^{x^2} (d) e^{-x}

9 જો $y = e^x$ હોય, તો $\frac{d^2y}{dx^2} = \dots\dots\dots$
 (a) e^x (b) e^{2x} (c) e^{x^2} (d) e^{-x}

8 If $x = \cos \theta$, $y = \sin \theta$ then $\frac{dy}{dx} = \dots\dots\dots$
 (a) $\cot \theta$ (b) $\tan \theta$ (c) $-\cot \theta$ (d) $-\tan \theta$

૮ જો $x = \cos \theta$, $y = \sin \theta$ તો $\frac{dy}{dx} = \dots\dots\dots$
 (અ) $\cot \theta$ (બ) $\tan \theta$ (ક) $-\cot \theta$ (ડ) $-\tan \theta$

9 $\frac{d}{dx} x^x = \dots\dots\dots$
 (a) $x - \log x$ (b) $x + \log x$ (c) $x^x(1 + \log x)$ (d) $x \cdot x^{x-1}$

૯ $\frac{d}{dx} x^x = \dots\dots\dots$
 (a) $x - \log x$ (b) $x + \log x$ (c) $x^x(1 + \log x)$ (d) $x \cdot x^{x-1}$

10 $\int \frac{1}{x^2} dx = \dots\dots\dots + C$
 (a) $\frac{1}{x}$ (b) $\frac{-1}{x}$ (c) $\frac{-1}{3x^3}$ (d) $\frac{1}{3x^3}$

૧૦ $\int \frac{1}{x^2} dx = \dots\dots\dots + C$
 (અ) $\frac{1}{x}$ (બ) $\frac{-1}{x}$ (ક) $\frac{-1}{3x^3}$ (ડ) $\frac{1}{3x^3}$

11 $\int \frac{1}{x^2 + 25} dx = \dots\dots + c$
 (a) $\tan^{-1}\left(\frac{x}{5}\right)$ (b) $\frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right)$ (c) $\frac{1}{5} \tan^{-1}\left(\frac{5}{x}\right)$ (d) $\tan^{-1}\left(\frac{5}{x}\right)$

૧૧ $\int \frac{1}{x^2 + 25} dx = \dots\dots + c$
 (અ) $\tan^{-1}\left(\frac{x}{5}\right)$ (બ) $\frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right)$ (ક) $\frac{1}{5} \tan^{-1}\left(\frac{5}{x}\right)$ (ડ) $\tan^{-1}\left(\frac{5}{x}\right)$

12 $\int_0^1 e^x dx = \dots\dots$
 (a) e-1 (b) 1-e (c) e (d) -e

૧૨ $\int_0^1 e^x dx = \dots\dots$
 (અ) e-1 (બ) 1-e (ક) e (ડ) -e

13 The order of a differential equation $\frac{d^2 y}{dx^2} = \left(3 + \frac{dy}{dx}\right)^3$ is
 (a) 3 (b) 2 (c) 1 (d) 6

૧૩ વિકલ્ સમીકરણ $\frac{d^2 y}{dx^2} = \left(3 + \frac{dy}{dx}\right)^3$ ની કક્ષા છે.
 (a) 3 (b) 2 (c) 1 (d) 6

14 The integrating factor(I.F.) of $\frac{dy}{dx} + \frac{2y}{x} = e^x$ is
 (a) $\frac{2}{x^2}$ (b) $\frac{2}{x^2}$ (c) x^2 (d) $2x$

૧૪ વિકલ્ સમીકરણ $\frac{dy}{dx} + \frac{2y}{x} = e^x$ નો સંકલ્પકારક અવયવ (I.F.) છે.
 (a) $\frac{2}{x^2}$ (b) $\frac{2}{x^2}$ (c) x^2 (d) $2x$

Q.2 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Find the modulus and principal argument of $z = \sqrt{3} + i$ and express z into polar form.

૧. $z = \sqrt{3} + i$ નો માનક અને કોણિક શોધો, તથા z ને ધ્રુવિય સ્વરૂપમાં અભિવ્યક્ત કરો

2. Find the square root of $3 - 4\sqrt{10}i$

૨. $3 - 4\sqrt{10}i$ નું વર્ગમૂળ શોધો

3. Prove that $(1 + \cos \theta + i \sin \theta)^n = 2^n \cos^n \left(\frac{\theta}{2}\right) \left[\cos \frac{n\theta}{2} + i \sin \frac{n\theta}{2} \right]$

3. સાબિત કરો કે $(1 + \cos \theta + i \sin \theta)^n = 2^n \cos^n \left(\frac{\theta}{2} \right) \left[\cos \frac{n\theta}{2} + i \sin \frac{n\theta}{2} \right]$

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

08

1. If $f(x) = \log x$, then prove that

(i) $f(x) + f(y) = f(xy)$ and (ii) $f(x) - f(y) = f(x/y)$

૧. જો $f(x) = \log x$ હોય, તો સાબિત કરો કે

(i) $f(x) + f(y) = f(xy)$ and (ii) $f(x) - f(y) = f(x/y)$

2. Evaluate : $\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3}$

૨. $\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3}$ મેળવો

3. Evaluate : $\lim_{x \rightarrow 0} \frac{x \log(1+x)}{1 - \cos x}$

૩. ત્રક્ષ શોધો $\lim_{x \rightarrow 0} \frac{x \log(1+x)}{1 - \cos x}$

Q.3 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Differentiate

e^x with respect to x using first principle of differentiation

૧. e^x નું વિકલન x ની સાપેક્ષ વિકલનનાં પ્રથમ સિધ્ધાંત થી કરો

2. Find $\frac{dy}{dx}$ if $y = \log(\sec x + \tan x)$

૨. જો $y = \log(\sec x + \tan x)$ તો $\frac{dy}{dx}$ મેળવો

3. Equation of motion of a particle is $s = t^3 - 6t^2 + 8t - 4$. Then find the velocity and acceleration of the moving particle at $t = 3$ second.

૩. કણની ગતિનું સમીકરણ $s = t^3 - 6t^2 + 8t - 4$ છે. તો $t = 3$ સેકન્ડે કણનો વેગ અને પ્રવેગ શોધો.

08

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

1. If $x^3 + y^3 = 3axy$ then find $\frac{dy}{dx}$

૧. જો $x^3 + y^3 = 3axy$ તો $\frac{dy}{dx}$ શોધો.

2. If $x = \frac{1}{2}\left(t + \frac{1}{t}\right)$ and $y = \frac{1}{2}\left(t - \frac{1}{t}\right)$ then find $\frac{dy}{dx}$.
૨. જો $x = \frac{1}{2}\left(t + \frac{1}{t}\right)$ અને $y = \frac{1}{2}\left(t - \frac{1}{t}\right)$ તો $\frac{dy}{dx}$ મેળવો.
3. Find the maximum and minimum values of $f(x) = 3x^3 - 4x^2 - x + 5$
૩. $f(x) = 3x^3 - 4x^2 - x + 5$ ની મહત્તમ અને ન્યૂનતમ મૂલ્યો શોધો.

Q.4 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Find $\int \cos x \cdot \sqrt{\sin x} dx$
૧. શોધો: $\int \cos x \cdot \sqrt{\sin x} dx$
2. Find $\int \frac{x}{(x+1)(x+2)} dx$
૨. શોધો : $\int \frac{x}{(x+1)(x+2)} dx$
3. Evaluate: $\int_0^{\frac{\pi}{2}} \log \cot x dx$
૩. કિંમત શોધો: $\int_0^{\frac{\pi}{2}} \log \cot x dx$

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

08

1. Find $\int x \cdot e^{3x} dx$
૧. શોધો : $\int x \cdot e^{3x} dx$
2. Evaluate: $\int_0^5 \frac{\sqrt[3]{x+2}}{\sqrt[3]{x+2} + \sqrt[3]{7-x}} dx$
૨. કિંમત શોધો: $\int_0^5 \frac{\sqrt[3]{x+2}}{\sqrt[3]{x+2} + \sqrt[3]{7-x}} dx$
3. Find the area bounded by the curve $y = x^2 - 7x + 10$ and x-axis .

૩. વક્ર $y = x^2 - 7x + 10$ અને X-અક્ષ વચ્ચે ઘેરાયેલા પ્રદેશનું ક્ષેત્રફળ શોધો

Q.5 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Evaluate : $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$

૧. મેળવો : $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$

2. Find the order and degree of $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \rho \left(\frac{d^2y}{dx^2}\right)^2$

૨. $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \rho \left(\frac{d^2y}{dx^2}\right)^2$ ની કક્ષા અને પરિમાણ મેળવો.

3. Form the differential equation whose general solution is $Y = A \cos x + B \sin x$

૩. જેનો સામાન્ય ઉકેલ $Y = A \cos x + B \sin x$ તે વિકલ સમીકરણ મેળવો.

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

08

1. Solve : $\frac{dy}{dx} = \frac{y}{x}$

૧. ઉકેલો : $\frac{dy}{dx} = \frac{y}{x}$

2. Solve $\frac{dy}{dx} = \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right)$.

૨. $\frac{dy}{dx} = \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right)$ ઉકેલો.

3. solve : $x \log x \frac{dy}{dx} + y = \log x^2$

૩. ઉકેલો : $x \log x \frac{dy}{dx} + y = \log x^2$
