

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY
DIPLOMA ENGINEERING – SEMESTER – II EXAMINATION – WINTER - 2018

Subject Code: 3320002

Date: 01-01-2019

Subject Name: ADVANCED MATHEMATICS (GROUP-1)

Time: 02:30 PM TO 05:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of programmable & Communication aids are strictly prohibited.
5. Use of only simple calculator is permitted in Mathematics.
6. English version is authentic.

Q.1 Fill in the blanks using appropriate choice from the given options. **14**

- | | | | | | |
|----------|--|---------------------|---------------------|------------------------------|--|
| 1 | $i^8 = \dots\dots\dots$ | | | | |
| | (a) 1 | (b) i | (c) -i | (d) 0 | |
| ૧ | $i^8 = \dots\dots\dots$ | | | | |
| | (a) 1 | (b) i | (c) -i | (d) 0 | |
| 2 | $Z + \bar{Z} = \dots\dots\dots$ | | | | |
| | (a) $-2\text{Re}(Z)$ | (b) $\text{Re}(Z)$ | (c) $2\text{Re}(Z)$ | (d) $2\text{Im}(Z)$ | |
| ૨ | $Z + \bar{Z} = \dots\dots\dots$ | | | | |
| | (a) $-2\text{Re}(Z)$ | (b) $\text{Re}(Z)$ | (c) $2\text{Re}(Z)$ | (d) $2\text{Im}(Z)$ | |
| 3 | If $Z = 2i$ then $\arg(Z) = \dots\dots\dots$ | | | | |
| | (a) $-\frac{\pi}{2}$ | (b) $\frac{\pi}{2}$ | (c) π | (d) 0 | |
| ૩ | જો $Z = 2i$ તો $\arg(Z) = \dots\dots\dots$ | | | | |
| | (a) $-\frac{\pi}{2}$ | (b) $\frac{\pi}{2}$ | (c) π | (d) 0 | |
| 4 | If $Z = \sqrt{3} - i$ then $\bar{Z}$ is $\dots\dots\dots$ | | | | |
| | (a) $-\sqrt{3} - i$ | (b) $\sqrt{3} + i$ | (c) $-\sqrt{3} + i$ | (d) $\frac{1}{\sqrt{3} - i}$ | |
| ૪ | જો $Z = \sqrt{3} - i$ તો $\bar{Z}$ $\dots\dots\dots$ | | | | |
| | (a) $-\sqrt{3} - i$ | (b) $\sqrt{3} + i$ | (c) $-\sqrt{3} + i$ | (d) $\frac{1}{\sqrt{3} - i}$ | |
| 5 | $\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \dots\dots\dots$ | | | | |
| | (a) $\log_e 2$ | (b) $\log_2 e$ | (c) 1 | (d) 0 | |
| ૫ | $\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \dots\dots\dots$ | | | | |
| | (a) $\log_e 2$ | (b) $\log_2 e$ | (c) 1 | (d) 0 | |
| 6 | $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = \dots\dots\dots$ | | | | |
| | (a) θ | (b) 1 | (c) 0 | (d) does not exists | |
| ૬ | $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = \dots\dots\dots$ | | | | |
| | (a) θ | (b) 1 | (c) 0 | (d) does not exists | |
| 7 | $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = \dots\dots\dots$ | | | | |
| | (a) $\frac{\pi}{2}$ | (b) 0 | (c) -1 | (d) 1 | |
| ૭ | $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = \dots\dots\dots$ | | | | |
| | (a) $\frac{\pi}{2}$ | (b) 0 | (c) -1 | (d) 1 | |

- 8 $\frac{d}{dx} (e^x) = \dots\dots\dots$
 (a) x (b) e (c) e^x (d) 0
- 9 $\frac{d}{dx} (e^x) = \dots\dots\dots$
 (a) x (b) e (c) e^x (d) 0
- 10 $\frac{d}{dx} (\tan^{-1} x) = \dots\dots\dots$
 (a) $\frac{1}{1+x^2}$ (b) $\frac{1}{1-x^2}$ (c) $\frac{-1}{1+x^2}$ (d) $\frac{-1}{1-x^2}$
- 11 $\frac{d}{dx} (\tan^{-1} x) = \dots\dots\dots$
 (a) $\frac{1}{1+x^2}$ (b) $\frac{1}{1-x^2}$ (c) $\frac{-1}{1+x^2}$ (d) $\frac{-1}{1-x^2}$
- 12 If $y = \log x$ then $\frac{d^2y}{dx^2} = \dots\dots\dots$
 (a) $\frac{1}{x}$ (b) $\frac{-1}{x^2}$ (c) $\frac{1}{x^2}$ (d) $\frac{-1}{x}$
- 13 જો $y = \log x$ તો $\frac{d^2y}{dx^2} = \dots\dots\dots$
 (a) $\frac{1}{x}$ (b) $\frac{-1}{x^2}$ (c) $\frac{1}{x^2}$ (d) $\frac{-1}{x}$
- 14 $\int \sin x \, dx = \dots\dots\dots$
 (a) $\cos x + c$ (b) $\sec x + c$ (c) $-\cos x + c$ (d) $\operatorname{cosec} x + c$
- 15 $\int \sin x \, dx = \dots\dots\dots$
 (a) $\cos x + c$ (b) $\sec x + c$ (c) $-\cos x + c$ (d) $\operatorname{cosec} x + c$
- 16 $\int_0^1 2x \, dx = \dots\dots\dots$
 (a) 0 (b) 1 (c) 2 (d) 3
- 17 $\int_0^1 2x \, dx = \dots\dots\dots$
 (a) 0 (b) 1 (c) 2 (d) 3
- 18 The order of a differential equation $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^3 = 0$ is $\dots\dots\dots$
 (a) 0 (b) 2 (c) 3 (d) 4
- 19 વિકલ સમીકરણ $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^3 = 0$ ક્રમ $\dots\dots\dots$ છે.
 (a) 0 (b) 2 (c) 3 (d) 4
- 20 An integrating factor (IF) of the equation $\frac{dy}{dx} + y = 3x$ is $\dots\dots\dots$
 (a) $\log x$ (b) 1 (c) 2 (d) e^x
- 21 વિકલ સમીકરણ $\frac{dy}{dx} + y = 3x$ નો સંકલ્પકારક અવયવ (IF) $\dots\dots\dots$ છે.
 (a) $\log x$ (b) 1 (c) 2 (d) e^x

Q.2 (a) Attempt any two કોઈપણ બે ના જવાબ આપો. <http://www.gujaratstudy.com>

06

- Find the square root of $7 + 24i$.
 ૧. $7 + 24i$ નું વર્ગમૂળ શોધો.
- If $Z = \operatorname{cis} \theta$, then show that $z^n + \frac{1}{z^n} = 2 \cos n\theta$ and $z^n - \frac{1}{z^n} = 2i \sin n\theta$.

૨. જો $Z = \operatorname{cis} \theta$, હોય તો સાબિત કરો કે $z^n + \frac{1}{z^n} = 2 \cos n\theta$ અને $z^n - \frac{1}{z^n} = 2i \sin n\theta$.

3. Find the modules and principal argument of $Z = \frac{1+i}{1-i}$, and express Z into Polar form.

૩. $Z = \frac{1+i}{1-i}$ નો માનક અને કોણિક શોધો. તથા Z ને ધ્રુવીય સ્વરૂપમાં અભિવ્યક્ત કરો.

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

08

- If $f(x) = \tan x$ then prove that (A) $f(x+y) = \frac{f(x) + f(y)}{1 - f(x)f(y)}$ (B) $f(2x) = \frac{2f(x)}{1 - [f(x)]^2}$

૧. જો $f(x) = \tan x$ હોય તો સાબિત કરો કે

$$(A) f(x+y) = \frac{f(x) + f(y)}{1 - f(x)f(y)}$$

$$(B) f(2x) = \frac{2f(x)}{1 - [f(x)]^2}$$

2. Find $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x^3 - 8}$

૨. $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x^3 - 8}$ ની કિંમત શોધો

3. Evaluate $\lim_{\theta \rightarrow 0} \frac{2 \sin \theta - \sin 2\theta}{\theta^3}$

૩. $\lim_{\theta \rightarrow 0} \frac{2 \sin \theta - \sin 2\theta}{\theta^3}$ ની કિંમત શોધો.

Q.3 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Differentiate $f(x) = \sqrt{x}$ with respect to x by using the definition of derivative.

૧. વ્યાખ્યાની મદદથી $f(x) = \sqrt{x}$ નું x - સાપેક્ષે વિકલન કરો.

2. If $y = \log \left(\frac{\sin x}{1 + \cos x} \right)$ then prove that $\frac{dy}{dx} = \cos x$.

૨. જો $y = \log \left(\frac{\sin x}{1 + \cos x} \right)$ હોય તો સાબિત કરો કે $\frac{dy}{dx} = \cos x$.

3. Find $\frac{dy}{dx}$, if $y = (\sin x)^x$

જો $y = (\sin x)^x$ હોય તો $\frac{dy}{dx}$ શોધો.

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

08

1. If $y = \log (\sin x)$ then prove that $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 + 1 = 0$.

૧. જો $y = \log (\sin x)$ હોય તો સાબિત કરો કે $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 + 1 = 0$.

2. The equation of motion of a particle is $S = t^3 - 3t^2 + 4t + 3$. Find its velocity and acceleration at $t = 2$ sec.

૨. કણની ગતિનું સમીકરણ $S = t^3 - 3t^2 + 4t + 3$ હોય તો $t = 2$ sec. આગળ કણનો વેગ અને પ્રવેગ શોધો.

3. Find maximum and minimum value of $y = x^3 - 3x + 11$.

૩. જો $y = x^3 - 3x + 11$ હોય તો અધિકતમ અને મહત્તમ કિંમત શોધો.

Q.4 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Evaluate $\int \frac{3+4 \sin x}{\cos^2 x} dx$

૧. સંકલન કરો $\int \frac{3+4 \sin x}{\cos^2 x} dx$

2. Evaluate $\int x \sin x dx$

૨. સંકલન કરો $\int x \sin x dx$

3. Evaluate $\int_1^2 (x^2 + 4x + 1) dx$

૩. સંકલન કરો $\int_1^2 (x^2 + 4x + 1) dx$

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

08

1. Evaluate $\int \frac{x}{(x+1)(x+2)} dx$
૧. સંકલન કરે $\int \frac{x}{(x+1)(x+2)} dx$
2. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$
૨. સંકલન કરે $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$

3. Find the area of the region bounded by $y = 2x^2$, X- axis and line $x = 5$.
૩. $y = 2x^2$ અને યુરેખા $x = 5$ થી ઘેરાયેલા પ્રદેશનું ક્ષેત્રફળ શોધો.

Q.5 (a) Attempt any two કોઈપણ બે ની જવાબ આપો.

06

1. Find $\lim_{n \rightarrow \infty} (\sqrt{n^2 + n + 1} - n)$
૧. $\lim_{n \rightarrow \infty} (\sqrt{n^2 + n + 1} - n)$ શોધો.
2. Find the differential equation for $y = a \sin(x + b)$, where a and b are arbitrary constant.
૨. વિકલ સમીકરણ $y = a \sin(x + b)$ નો ઉકેલ શોધો. જ્યાં a અને b અચળ છે.
3. Solve the differential equation $\tan y dx + \tan x \sec^2 y dy = 0$.
૩. વિકલ સમીકરણ $\tan y dx + \tan x \sec^2 y dy = 0$ નો ઉકેલ શોધો.

(b) Attempt any two કોઈપણ બે ની જવાબ આપો.

08

1. Solve the differential equation $\frac{dy}{dx} = (x + y)^2$
૧. વિકલ સમીકરણ $\frac{dy}{dx} = (x + y)^2$ નો ઉકેલ શોધો.
2. Solve: $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) - x^2$.
૨. ઉકેલો : $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) - x^2$
3. Solve : $\frac{dy}{dx} + y \tan x = \sec x$
૩. ઉકેલો : $\frac{dy}{dx} + y \tan x = \sec x$
