Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER–I • EXAMINATION – SUMMER • 2014

Subject Code: 2610003 **Date: 18-06-2014**

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 10:30 am - 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define poset. When is a poset said to be a lattice? Draw Hasse diagrams of following posets and examine which of them are lattices. **07**

(a) $< P(S), \subseteq >$, S = {a, b, c}

(b) < {1, 2, 3, 12, 18}, D>

(c) < {1, 2, 3, 6}, D>

(d) $< S_{16}$, D>.

(b) (1) Show that a lattice with three or fewer elements is a chain. **03**

(2) Find the complements of every element of the lattice $< S_n$, D> for n = 45. **04**

Q.2 **(a)** (1) Show that in a lattice if $a \leq b \leq c$ then **03**

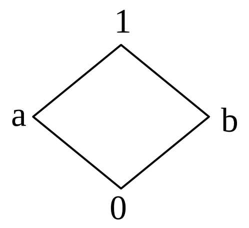
(i) $a \oplus b = b * c$

(ii) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$.

(2) Define a Distributive lattice. Prove that in a distributive lattice, complement of an element, if it exists, is unique. **04**

(b) (1) Find the value of $x_1 * x_2 * [(x_1 * x_4) \oplus x_2' \oplus (x_3 * x_1')]$ **03**

for $x_1 = a$, $x_2 = 1$, $x_3 = b$ and $x_4 = 1$ where $a, b, 1 \in B$ and the Boolean algebra $< B, *, \oplus, ', 0, 1>$ is given by



(2) Obtain the sum-of-products canonical form of the following Boolean expressions: **04**

(i) $(x_1 \oplus x_2)' \oplus (x_1' * x_3)$

(ii) $(x_1 * x_2') \oplus x_3$

OR

(b) Use the Quine-McCluskey algorithm to find the prime implicants of the expression: **07**

$f(a, b, c, d) = \sum (0, 1, 4, 5, 9, 11)$. Also obtain a minimal expression for the same.

Q.3 **(a)** (1) Prove that the only idempotent element in a group is the identity element. **03**

(2) Define: Abelian group, Cyclic group. Show that every cyclic group is abelian. Is the converse true? Justify your answer. **04**

(b) Define: Kernel of a group homomorphism. Show that it is a subgroup. **07**

OR

Q.3 **(a)** (1) Show that in a group $<G, *>$, if for any a, b $\in$ G, $(a * b)^2 = a^2 * b^2$, then $<G, *>$ must be abelian. **03**

(2) Show that $<\{1,4,13,16\}, \times_{17}>$ is a subgroup of $<Z_{17}^*, \times_{17}>$. **04**

(b) Prove that $< Z_7^*, \times_7 >$ is a group. What are the generators of this group? **07**

Q.4 **(a)** (1) Show that statement formula A logically implies statement formula B where A: $\sim q \wedge (p \wedge q)$ and B: $\sim p$. **03**

(2) Let P(x, y) denote the sentence: $2x + y = 1$.What are the truth values of $\forall x \exists y P(x, y)$, $\forall x \forall y P(x, y)$ and $\exists x \exists y P(x,y)$ where the domain of x, y is the set of all integers? **04**

(b) (1) Construct the truth table for each of the following statement formulas. **04**

(i) $(p \rightarrow q \wedge r) \vee (\sim p)$

(ii) $(p \vee q) \leftrightarrow (q \rightarrow r)$.

(2) Show without constructing the truth table that the statement formula $\sim p \rightarrow (p \rightarrow q)$ is a tautology. **03**

OR

Q.4 **(a)** (1) Symbolize the following sentences by using predicates, quantifiers and logical connectives. **04**

(i) Every integer is either odd or even.

(ii) If you buy a car, then you must pay a sales tax.

(iii) Some people are vegetarians.

(2) Give an indirect proof to show that if $n^2 + 3$ is odd then n is even. **03**

(b) (1) Prove that $\forall$ x $\in$ Z, $x^2 - x$ is an even integer. **04**

(2) Test the validity of the following argument: **03**

If it snows, then the streets become slippery. If the streets become slippery, then accidents happen. Accidents do not happen. Therefore, it does not snow.

Q.5 **(a)** Define Node base. State properties of node base. Explain why no node in a node base is reachable from another node in the node base. **07**

(b) From the adjacency matrix of a simple digraph, how will you determine whether it is a directed tree? If it is a directed tree, how will you determine its root and terminal nodes? **07**

OR

Q.5 **(a)** Define strongly connected graph. Show that in a simple digraph G = < V, E >, every node of the digraph lies in exactly one strong component. **07**

(b) Define: Directed tree and its leaf. Draw the graph of the tree represented by (A(B(C(D)(E)))(F(G)(H)(J))(K(L)(M)(N(P)(Q(R))))). Obtain the binary tree corresponding to it. **07**

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