

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER- 1 EXAMINATION – SUMMER 2016

Subject Code: 2610003

Date: 09-05-2016

Subject Name: Discrete Mathematics for Computer Science

Time: 02.30pm to 05.00 pm

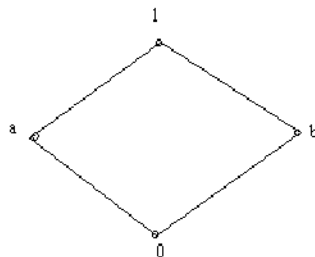
Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Define distributive lattice. Prove that every chain is distributive lattice. Prove that the power set lattice is a complemented distributive lattice. **07**
- (b) 1) Give an example of a relation which is neither reflexive nor irreflexive. **02**
 2) Give an example of a relation which is both symmetric and antisymmetric **02**
 3) Given $S = \{1, 2, \dots, 10\}$ and a relation R on S where $R = \{ \langle x, y \rangle / x + y = 10 \}$ what are the properties of the relation R ? **03**

- Q.2** (a) 1) Determine the truth value of each statement given below. The domain of discourse is the set of real numbers. Justify your answers. **03**
 i) For every x , $x^2 > x$
 ii) For some x , $x^2 > x$
 iii) For every x , if $x > 1$ then $x^2 > x$.
 2) Prove using Indirect proof technique that if $n^2 + 3$ is odd then n is even. **04**
- (b) 1) Given an expression $\alpha(x_1, x_2, x_3)$ defined to be $\sum(0, 3, 5, 7)$, determine the value of $\alpha(a, b, 1)$, where $a, b, 1 \in B$ and $\langle B, *, \oplus, ', 0, 1 \rangle$ is a Boolean algebra given in the following figure. **04**



- 2) Define poset, lattice and chain. Give one example of each with your justification. Draw their Hasse diagrams also. **03**

OR

- (b) 1) Draw the Hasse diagram of the following lattices. $\langle S_{100}, D \rangle$, $\langle S_{36}, D \rangle$, $\langle S_4 \times S_{25}, D \rangle$, $\langle S_6 \times S_6, D \rangle$. Which of them are isomorphic? **04**
 2) Define complemented lattice. Find the compliments of every elements of the lattice $\langle S_n, D \rangle$ for $n = 75$. **03**
- Q.3** (a) Define Boolean function. Use the Karnaugh map representation to find a minimal sum of products expression of the following function : **07**
 $f(x_1, x_2, x_3, x_4) = \sum(14, 13, 11, 5, 2, 1)$
- (b) 1) Let $(B, *, \oplus, ', 0, 1)$ be a Boolean algebra prove the following : **04**
 $a = b \Leftrightarrow (a * b') \oplus (a' * b) = 0$
 2) Obtain the sum of product canonical form of Boolean expression in three variables x_1, x_2, x_3 for $(x_1 \oplus x_2) * x_3$ **03**

OR

- Q.3** (a) Define “Universal quantifier” and “Existential quantifier”. **07**
 (1) Formulate the symbolic expression for
 i) $p \rightarrow q$

ii) $(p \vee q)' \leftrightarrow r$

in words using:

p : Today is Monday

q : It is raining

r : It is hot.

(2) State the rule UG in the predicate calculus. Verify whether the following conclusion is valid or not.

$$(x)(P(x) \rightarrow Q(x)), \neg Q(a) \Rightarrow (x) \neg P(x)$$

- (b) Use the Quine-McClusky algorithm to obtain the minimal SOP form of the function $f(a, b, c, d) = \sum(0, 2, 5, 7, 8, 10, 13, 15)$ **07**

Q.4 (a) Define cyclic group. Show that cyclic group is abelian but converse is not true. **07**
Is $\langle \mathbb{Z}_5, +_5 \rangle$ a cyclic group? If so, find its generators.

- (b) Define symmetric group $\langle S_3, \diamond \rangle$. Write all its elements and composition table. **07**
Show that it is non-abelian. Determine all the proper subgroups of $\langle S_3, \diamond \rangle$.

OR

Q.4 (a) Define left coset of a subgroup $\langle H, * \rangle$ in the group $\langle G, * \rangle$. Find left cosets of $\{[0], [3]\}$ in the group $\langle \mathbb{Z}_6, +_6 \rangle$. **07**

- (b) Define kernel of a group homomorphism. If $\langle G, * \rangle$ and $\langle H, \Delta \rangle$ are two group and $g: G \rightarrow H$ is a homomorphism, show that $\ker(g)$ is a normal subgroup of $\langle G, * \rangle$. **07**

Q.5 (a) Give an abstract definition of graph. When are two simple graphs said to be isomorphic? Give an example of two simple digraphs having 4 nodes and 4 edges which are not isomorphic. **07**

- (b) Define a directed tree. Draw the graph of the tree represented by $(A(B(E(H)(I))(F(J)(K))(G(L)))(C(M(O))(N(P)(Q)))(D(R(S(V))(T)(U))))$ **07**
Obtain the binary tree corresponding to it.

OR

Q.5 (a) 1) Define Cyclic graph, Null graph, and Strongly connected graph. **03**

2) Define Adjacency matrix and path matrix of a graph. Explain each with example. **04**

- (b) Define nodebase of a simple digraph. Find the reachability set of all nodes for the digraph given in following figure. Also find the nodebase for it. **07**

