

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER-I • EXAMINATION – WINTER • 2014

Subject Code: 2610003

Date: 30-12-2014

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 10:30 am - 01:00 pm

Total Marks: 70

Instructions:

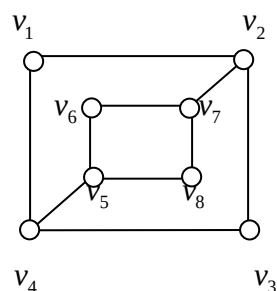
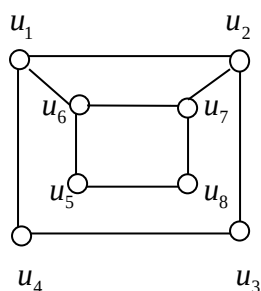
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** (I) Given $S = \{1, 2, 3, 4, 5\}$ and a relation R on S where $R = \{(x, y) / x + y = 5\}$, **03**
What are the properties of the relation R ?
(II) Let $X = \{1, 2, 3, 4\}$ and $R = \{(x, y) / x > y\}$ **04**
(i) Draw graph of R . (ii) Write its matrix.
- (b)** (I) Define a group. Show that $(I, +)$, where I is the set of integers and $+$ is Operation of usual addition, is a group. **03**
(II) Define a subgroup. Show that $\langle \{[1], [4], [13], [16]\}, \times_{17} \rangle$ is a subgroup of $\langle Z_{17}^*, \times_{17} \rangle$ **04**
- Q.2 (a)** (I) State whether following lattices are distributive as well as complemented or not. Justify your answers. **03**
(i) (S_{16}, D) (ii) (S_{45}, D)
(II) Draw Hasse diagram of following Posets. **04**
(i) $(\rho(A), \subseteq)$ where $A = \{a, b, c\}$
(ii) (S_{36}, D)
- (b)** (I) Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(x, y) / x - y \text{ is divisible } 3\}$ **04**
Show that R is an equivalence relation.
(II) Give an example of a set X such that $\langle \rho(X), \subseteq \rangle$ is a chain. **03**
- OR**
- (b)** (I) Given a set $S = \{1, 2, 3, 4, 5\}$. Find the equivalence relation on S , which generates the partition $\{\{1, 2\}, \{3\}, \{4, 5\}\}$. **03**
(II) Let $R = \{(1, 2), (3, 4), (2, 2)\}$ and $S = \{(4, 2), (2, 5), (3, 1), (1, 3)\}$, **04**
Find $R \circ S$, $S \circ R$, $R \circ R$, $S \circ S$.
- Q.3 (a)** (I) Determine the truth value of each of the following statements. **03**
(i) $72 > 15$ and 33 is a prime integer.
(ii) $19 - 4 = 15$ or today's temperature is below freezing.
(iii) If Mickey is in Florida, then 17 is an odd integer.
(II) Test the validity of the logical consequences: **04**
All dogs fetch.
Kitty does not fetch.
Therefore Kitty is not a dog.
- (b)** (I) Define a cyclic group. Show that $(Z_6, +_6)$ is a cyclic group. **02**
(II) Define (i) A group homomorphism (ii) A group isomorphism. **05**
Show that $(Z_4, +_4)$ is isomorphic to $(Z_5^*, \times_5)$
- OR**
- Q.3 (a)** (I) Show by truth table that the following statement formula is a tautology: **03**
 $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

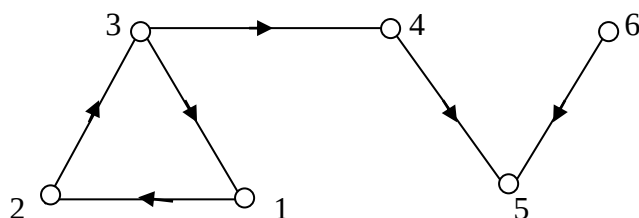
- (II) Test the validity of the logical consequences: 04
 All men are mortal.
 Randy is a man.
 Therefore Randy is mortal.
- (b) (I) Define an abelian group. Show that in a group $(G, *)$, if for any 03
 $a, b \in G$, $(a * b)^2 = a^2 * b^2$ then $(G, *)$ must be abelian. 04
 (II) Define kernel of a group homomorphism. Show that the kernel of Homomorphism g is a subgroup of $(G, *)$, where g is a homomorphism from $(G, *)$ to (H, Δ) .
- Q.4 (a)** (I) Show that in a lattice $(L, \leq)$, for any $a, b \in L$, $a \leq b \Leftrightarrow a \oplus b = b$ 03
 (II) Define a sublattice. Find all sublattices of (S_{10}, D) 04
- (b) (I) Show that the following Boolean expressions are equivalent. Obtain their sum-of-products canonical form. 03
 (i) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$
 (ii) $(x \oplus y) * (x' \oplus z)$
 (II) Use the Quine - McCluskey method to simplify the sum-of-products expression: 04

$$f(a, b, c, d) = \sum(10, 12, 13, 14, 15)$$
- OR**
- Q.4 (a)** (I) Show that in a lattice $(L, \leq)$, for any $a, b \in L$, $a \leq b \Leftrightarrow a * b = a$ 03
 (II) Define a Boolean Algebra. Show that in a Boolean Algebra, 04
 $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$
- (b) (I) Define (i) A join-irreducible element (ii) A meet-irreducible element in a Boolean Algebra. Write all join-irreducible elements and meet-irreducible elements of (S_{30}, D) . 04
 (II) Use the Karnaugh map representation to find a minimal sum-of-products expression of following function. 03

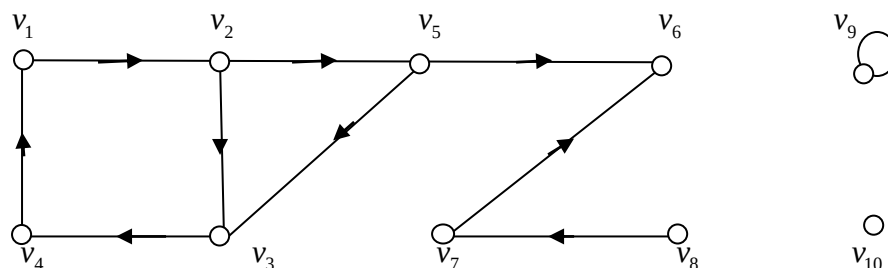
$$f(a, b, c, d) = \sum(0, 1, 2, 3, 12, 14)$$
- Q.5 (a)** (I) Define a path in graph. Define length of the path. What is difference between a simple path and an elementary path? 03
 (II) Define isomorphic graphs. State whether the following graphs are isomorphic or not. 04



- (b) (I) Define a strong component. Write strong components of the digraph given in following figure. 03

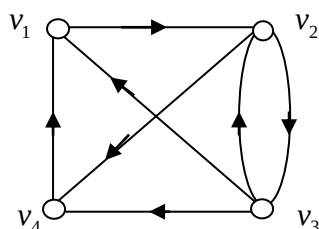


- (II) Define a node base. Write node base of the digraph given in following digraph. 04

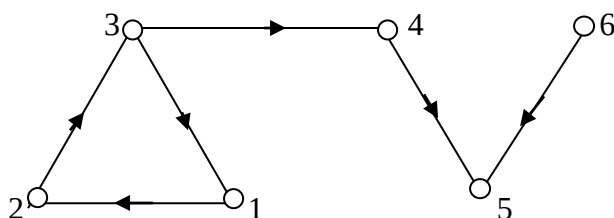


OR

- Q.5 (a) (I) Define adjacency matrix of the digraph G . Write adjacency matrix of the following digraph. 04



- (II) Define a unilateral component. Write unilateral components of the Graph given in following figure. 03



- (b) (I) Define a complete binary tree. What will be number of terminal nodes in a complete binary tree having 8 edges? 04
 (II) Define (i) An m -ary tree. (ii) An ordered tree (iii) A positional m -ary 03
