Seat No.: ________ Enrolment No.________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**MCA - SEMESTER–I • EXAMINATION – WINTER • 2015**

**Subject Code: 2610003** **Date: 30-12-2015**

**Subject Name: Discrete Mathematics for Computer Science**

**Time: 10:30 am - 01:00 pm** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Let $X = \{1,2,3,4\}$ and $R = \{(x, y) / x > y\}$ be relation on it.

(i) Write properties of $R$ **03**

(ii) Write matrix of $R$ **02**

(iii) Draw graph of $R$ **02**

**(b)** (I) Define a group. **01**

Let $Z$ be set of integers. (i) Is $(Z,+)$ a group? Justify your answer. **02**

(ii) Is $(Z,\times)$ a group? Justify your answer. **02**

(II) Define a cyclic group. Show that a cyclic group is always abelian. **02**

**Q.2** **(a)** (I) Define a partial order relation. Let $A$ be a finite set and $\rho(A)$ be its power Set. Show that $\subseteq$ ( set inclusion ) is a partial order relation on $\rho(A)$. **04**

(II) Define $R$ - equivalence classes. Let $I$ be the set of integers and $R$ be the relation "congruence modulo 3". Determine the equivalence classes generated by the elements of $I$. **03**

**(b)** (I) Draw Hasse' Diagram of the following posets. **04**

(i) $(S_{75}, D)$ (ii) $(S_{27}, D)$

(II) Let $R = \{(1,2), (3,4), (2,2)\}$ and $S = \{(4,2), (2,5), (3,1), (1,3)\}$. **03**

Find $R \circ S$, $S \circ R$, and $R \circ R$

**OR**

**(b)** (I) In poset $(S_{36}, D)$, find (i) GLB X, LUB X (ii) GLB Y, LUB Y where X = { 4, 6, 12 } and Y = { 3, 6, 9 } . **04**

(II) Write a short note on applications of relations to database theory. **03**

**Q.3** **(a)** (I) Determine the truth value of each of the following statements. **02**

(i) 72 > 15 and 33 is a prime integer.

(ii) If Anil is in America, then 19 is a prime integer.

(II) Write existential quantification of the sentence: **02**

" x is a prime integer, where, x is an odd integer."

Is this existential quantification a true statement?

(III) Test the validity of the logical consequences: **03**

All dogs fetch.

Ketty does not fetch.

Therefore, Ketty is not a dog.

**(b)** (I) Define a subgroup. What is the relation between order of a subgroup and order of a finite group? Find all the subgroups of $(Z_7^*, \times_7)$. **04**

(II) Define a left coset of a subgroup in a group. Find the left cosets of $\{[0],[3]\}$ in the group $(Z_6, +_6)$. **03**

**OR**

**Q.3** **(a)** (I) Determine the truth value of each of the following statements. **02**

(i) Today is Monday or 17 is an odd integer

(ii) If 4+5=10, then $16 \times 16 = 512$

(II) Write universal quantification of the sentence: **02**

"$x^2 + x$ is an even integer, where $x$ is an even integer."

Is this universal quantification a true statement?

(III) Test the validity of the logical consequences: **03**

Every integer is a rational number.

3 is an integer.

Therefore, 3 is a rational number.

**(b)** (I) Define a subgroup. Find the subgroup of symmetric group $S_4$ generated by the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$. **03**

(II) Show that if a group $(G, *)$ is of even order, then there must be an element $a \in G$ such that $a \neq e$ and $a * a = e$. **04**

**Q.4** **(a)** (I) Define (i) A complemented lattice **04**

(ii) A distributive lattice

Give one illustration for each

(i) A bounded lattice which is complemented but not distributive.

(ii) A bounded lattice which is distributive but not complemented.

(II) Show that in a complemented distributive lattice, **03**

$a \le b \Leftrightarrow b' \le a'$.

**(b)** (I) Define atoms and anti-atoms of a Boolean algebra. What is relation between Atoms and anti-atoms? Write atoms and anti-atoms of Boolean algebra $(\rho(S), \cap, \cup, \sim, \phi, S)$ where $S = \{a, b, c\}$ **03**

(II) Use Karnaugh map representation to find a minimal sum-of-products expression of function **04**

$f(a,b,c,d) = \sum(0,2,6,7,8,9,13,15)$.

**OR**

**Q.4** **(a)** (I) Show that De Morgan's laws hold true in a complemented, distributive lattice. **04**

(II) Define a sublattice. Give any four sublattices of the lattice $(S_{12}, D)$. **03**

**(b)** (I) Write the Boolean expression $x_1 * x_2$ in an equivalent sum - of- products Canonical form in three variables $x_1, x_2$ and $x_3$. **03**

(II) Use the Quine Mc Clusky method to simplify the sum-of-products expression **04**

$f(a,b,c,d) = \sum(0,2,4,6,8,10,12,14)$.

**Q.5** **(a)** (I) Define (i) The adjacency matrix of a graph G. **02**

(ii) The path matrix of a graph G.

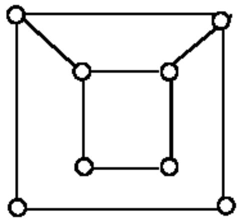
(II) Define (i) A unilaterally connected graph. **02**

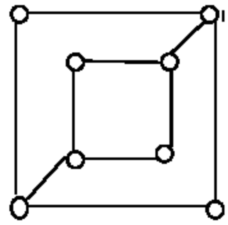
(ii) A strongly connected graph.

(III) Give a directed tree representation of the following formula. **03**

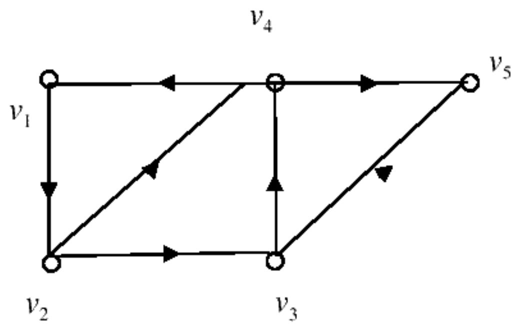
$(v_0(v_1(v_2)(v_3(v_4)(v_5)))(v_6(v_7(v_8))(v_9)(v_{10})))$.

**(b)** (I) Define isomorphic graphs. State whether the following digraphs are isomorphic or not. Justify your answer. **04**





(II) Find the reachable sets of $\{v_1, v_4\}, \{v_4, v_5\}$ and $\{v_3\}$ for the digraph given Below. **03**




**OR**

**Q.5** **(a)** (I) Define node base of a simple digraph. Comment upon statements: **04**

(i) No node in the node base is reachable from another node in the node base.
(ii) Any node whose indegree is zero must be present in any node base.
(iii) Any node that does not have indegree zero and does not lie on a cycle cannot be present in a node base.

(II) Define a complete binary tree. Show that in a complete binary tree, the total number of edges is $2(n_i - 1)$, where $n_i$ is the number of terminal nodes. **03**

**(b)** (I) Define the adjacency matrix of a graph G. Write adjacency matrix for the Following cases. **03**

(i) $G(V, E)$ where $V = \{v_1, v_2, \ldots v_7\}$ and $E = \phi$.
(ii) $G(V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and

$$E = \{(v_1, v_1), ((v_2, v_2), (v_3, v_3), (v_4, v_4), (v_5, v_5)\}.$$

(II) Define isomorphic graphs. What are the necessary conditions for two Graphs to be isomorphic? Are they sufficient also? Justify your answer. **04**

*************