Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER– I- EXAMINATION – WINTER - 2016

Subject Code: 2610003 **Date:03/01/ 2017**

Subject Name: Discrete Mathematics for computer science

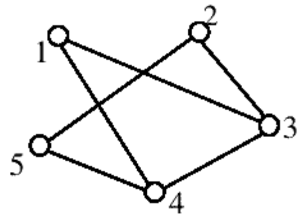
Time:10.30 AM TO 01.00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Define a binary relation. Let $X = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(x, y) / x + y = 6\}$ be a relation defined on X, write properties of R. Justify your answer. **04**

(ii) Define a maximal compatibility block. Write maximal compatibility blocks of the compatibility relation given in following figure. **03**



(b) (i) Define a group. Show that $(Z_7, +_7)$ is a group. Write all the subgroups of $(Z_7, +_7)$. **04**

(ii) Show that $(\{[1], [4], [13], [16]\}, \times_{17})$ is a subgroup of $(Z_{17}^*, \times_{17})$. **03**

Q.2 **(a)** (i) Define an equivalence relation. Let $X = \{1, 2, 3, 4\}$ and $R = \{(1,1), (1,4), (4,1), (4,4), (2,3), (3,2), (2,2), (3,3)\}$. Is R an equivalence Relation on X ? Justify your answer. **03**

(ii) Define a partial order relation. Let $A = \{a, b, c\}$ and $\rho(A)$ be its power set. Show that $(\rho(A), \subseteq)$ is a partial ordered set, where $\subseteq$ denotes relation of set-inclusion. **04**

(b) (i) Draw Hasse Diagram of the following posets **04**

(I) (S_{24}, D) (II) (S_{70}, D)

(ii) Write the order and the degree of the group $(S_n, \lozenge)$, where S_n is the set of all permutations of n elements and $\lozenge$ denotes the operation of right composition of permutations. Also write the order of the group $(D_n, \lozenge)$, where D_n is the set of all rigid rotations of a regular polygon of n sides under the same operation $\lozenge$. **03**

OR

(b) Define direct product of two lattices. Show that the lattice (S_{36}, D) is isomorphic to the direct product of lattices (S_4, D) and (S_9, D), where S_n is the set of positive divisors of natural number n and D denotes relation " to be divisor of". **07**

Q.3 **(a)** (i) Let P and Q be two statements. Write (I) converse (II) contra positive and (III) inverse statements of implication $P \rightarrow Q$. **03**

(ii) Determine the truth value of each of the following statement. **02**

(I) Either today is Monday or 7 is a real number.

(II) If Mickey is in Florida, then 17 is an odd integer.

(iii) Show without constructing the truth table that the statement formula $(\sim p) \rightarrow (p \rightarrow q)$ is a tautology **02**

(b) (i) Define (I) A group homomorphism (II) Kernel of group homomorphism. Show that kernel of a homomorphism g from a group $(G, *)$ to (H, Δ) is never an empty set. **04**

(ii) Define a normal subgroup. Show that every subgroup of a cyclic group is normal. **03**

OR

Q.3 **(a)** (i) Determine the truth value of each statement given below. The domain of discourse is the set of real numbers. Justify your answers. **03**

(I) For every x, $x^2 \geq x$.

(II) For some x, $x^2 > x$.

(III) For every x, $x^2 \geq 0$

(ii) Test the validity of the logical consequences: **04**

All dogs fetch.

Ketty does not fetch.

Therefore, Ketty is not a dog.

(b) (i) Find all subgroups of (I) $(Z_{12}, +_{12})$ and (II) $(Z_{11}^{*}, \times_{11})$ **04**

(ii) Define left coset of a subgroup $(H, *)$ in a group $(G, *)$ determined by the element $a \in G$ .Find the left cosets of $\{[0],[3]\}$ in the group $(Z_6, +_6)$. **03**

Q.4 **(a)** (i) Define atoms and anti-atoms of a Boolean algebra. What is relation between atoms and anti-atoms? Write atoms and anti-atoms of Boolean algebra $(p(S), \cap, \cup, \sim, \emptyset, S)$ where $S = \{x, y, z\}$ **04**

(ii) Obtain the sum of product canonical form of Boolean expression $(x_1 \oplus x_2) * x_3$ in three variables x_1, x_2, x_3. **03**

(b) (i) Define a Boolean Algebra. Show that in a Boolean Algebra, **03**

$a \leq b \Leftrightarrow a * b' = 0$

(ii) Use Karnaugh map representation to find a minimal sum-of-products expression of function **04**

$f(a,b,c,d) = \sum (0, 2, 6, 7, 8, 9, 13, 15)$

OR

Q.4 **(a)** (i) Give one illustration for each of the following: **04**

(I) A complemented and distributive lattice.

(II) A distributive but not a complemented lattice.

(III) A complemented but not a distributive lattice.

(IV) A lattice which is neither complemented nor distributive.

(ii) Show that **03**

$$(x_1' * x_2' * x_3' * x_4') \oplus (x_1' * x_2' * x_3' * x_4) \oplus (x_1' * x_2' * x_3 * x_4) \oplus (x_1' * x_2' * x_3 * x_4') = x_1' * x_2'$$

(b) (i) Define a complemented lattice. Which of the two lattices (S_n, D) for n = 42 and n = 45 are complemented? Justify your answer. **03**

(ii) Use the Quine - McCluskey method to simplify the sum-of-products expression **04**

$f(a,b,c,d) = \sum(0,3,4,7,8,11,12,15)$

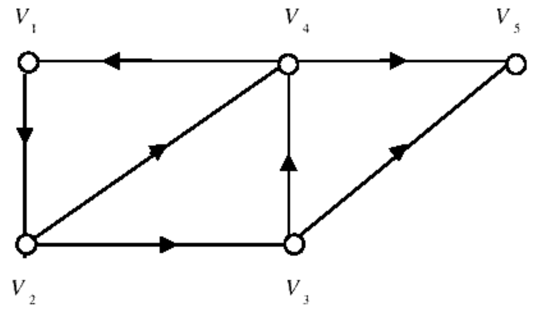
Q.5 **(a)** (i) Define node base of a simple digraph. Comment upon statements: **03**

(I) No node in the node base is reachable from another node in the node base.

(II) Any node whose indegree is zero must be present in any node base.

(ii) Define Isomorphic graphs. What are the necessary conditions for two graphs to be isomorphic? Are they sufficient also? Justify your answer **04**

(b) (i) Find the reachable sets of $\{v_1\}, \{v_3\}$ and $\{v_5\}$ for the digraph given below: **03**



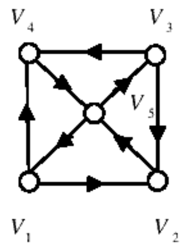
(ii) Define a complete binary tree. Show that in a complete binary tree the total number of edges is given by $2(n_t - 1)$, where n_t is the number of terminal nodes. **04**

OR

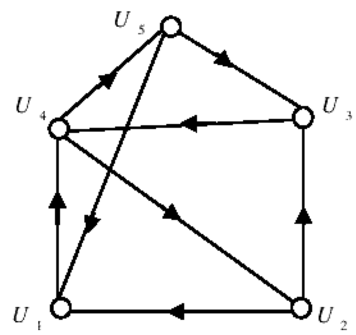
Q.5 **(a)** (i) Define a node base of a simple digraph. Prove that in an acyclic simple digraph a node base consists of only these nodes whose indegrees is zero. **04**

(ii) Define a directed tree. Show by means of an example that a simple digraph in which exactly one node has indegree 0 and every other node has indegree 1 is not necessarily a directed tree. **03**

(b) (i)) Define isomorphic graphs. State whether the following digraphs are Isomorphic or not. Justify your answer. **04**



(I)



(II)

(ii) Define : (I) A complete m-ary tree.

(II) A descendent of a node u in a directed tree. **03**

(III) A son of a node u in a directed tree.

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