Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER– I • EXAMINATION – WINTER - 2017

Subject Code: 2610003 Date: 01-01-2018

Subject Name: Discrete Mathematics for Computer Science

Time: 10:30 am to 01:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Let $X=\{a,b,c\}$. Give one example of each of the following: **04**

(i) A binary relation R on X, which is neither reflexive nor irreflexive

(ii) A binary relation S on X, which is symmetric and antisymmetric both.

(2) Let R= $R=\{(1,2),(3,4),(2,2)\}$ and $S=\{(4,2),(2,5),(3,1),(1,3)\}$, **03**

Find (i) $R\circ S$ (ii) $S\circ R$ (iii) $S\circ S$

(b) Define a partially ordered set. Draw Hasse' Diagram for the following posets **07**

(i) (S_{16},D) (ii) (S_{30},D) (iii) (S_{80},D)

Q.2 **(a)** Define a group. Let I be the set of integers and $+$, $\times$ denote usual operations of addition and multiplication respectively. **07**

(i) Is $(I,+)$ a group? Justify your answer.

(ii) Is $(I,\times)$ a group? Justify your answer.

(b) (1) Give one illustration for each of the following: **04**

(i) A complemented and distributive lattice.

(ii) A distributive but not a complemented lattice.

(iii) A complemented but not a distributive lattice.

(iv) A lattice which is neither complemented nor distributive.

(2) Write a short note on applications of relations to database theory **03**

OR

(b) Define direct product of two lattices. Show that the lattice (S_{36},D) is isomorphic to the direct product of lattices (S_4,D) and (S_9,D), where S_n denotes the set of all positive divisors of natural number n and D denotes the binary relation of "to be divisors of" **07**

Q.3 **(a)** (1) Show by constructing truth table that the following statement formula is a Tautology **04**

$((p\rightarrow q)\wedge(q\rightarrow r))\rightarrow(p\rightarrow r)$

(2) Test whether the given argument is logically valid: **03**

"If it rains, the prices of vegetables go up. The prices of vegetables go up. So it rains."

(b) Define (i) a group homomorphism (ii) Kernel of group homomorphism. **07**

Let g be a homomorphism from a group $(G,*)$ to a group (H,Δ)

. Show that

$g(e_G)=e_H$ and for any $a\in G$, $g(a^{-1})=(g(a))^{-1}$

OR

Q.3 **(a)** (1) Show by constructing truth table that $\sim(p \vee q) \leftrightarrow (\sim p) \wedge (\sim q)$ **04**

03

(2) Test whether the given argument is logically valid:

"If I save money, I will buy a house. I did not buy a house. Therefore I had not saved money."

(b) Define a subgroup. Let $(H_1, *)$ and $(H_2, *)$ be subgroups of a group $(G, *)$. Show that $(H_1 \cap H_2, *)$ is also a subgroup of $(G, *)$. Show that in general, $(H_1 \cup H_2, *)$ is not a subgroup of $(G, *)$, except when $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$. **07**

Q.4 **(a)** (1)Define a complemented lattice. Find the complements of every element in the lattice (S_{70}, D). **04**

(2) Show that in a lattice, if $a \leq b$ and $c \leq d$ then $a * c \leq b * d$. **03**

(b) Use Karnaugh map representation to find a minimal sum-of-products expression of each of the following functions. **07**

(i) $f(a,b,c) = \sum(0, 1, 4, 6)$

(ii) $f(a,b,c,d) = \sum(0, 5, 7, 8, 12, 14)$

OR

Q.4 **(a)** (1) Define a Boolean Algebra. Show that in a Boolean algebra, **04**

$a = b \Leftrightarrow ab' + a'b = 0$

(2) Show that the Boolean expressions (i) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$ **03**

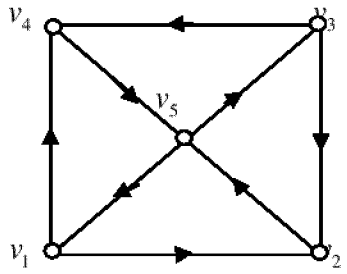
and (ii) $(x * z) \oplus (x' * y)$ are equivalent to each other.

(b) Use Quine - McCluskey method to find a minimal sum-of-products expression of each of the following functions: **07**

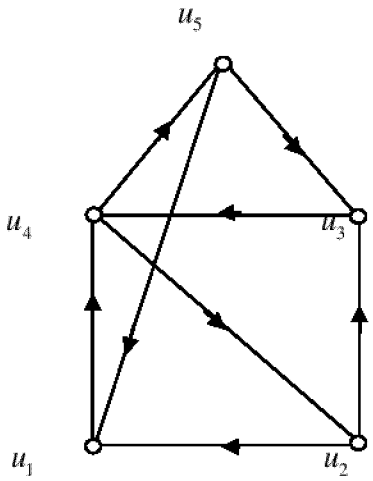
(i) $f(a, b, c, d) = \sum(0, 1, 2, 3, 13, 15)$

(ii) $f(a, b, c, d) = \sum(10, 12, 13, 14, 15)$

Q.5 **(a)** (1) Define isomorphic graphs. State whether the following digraphs are isomorphic or not. Justify your answer. **04**

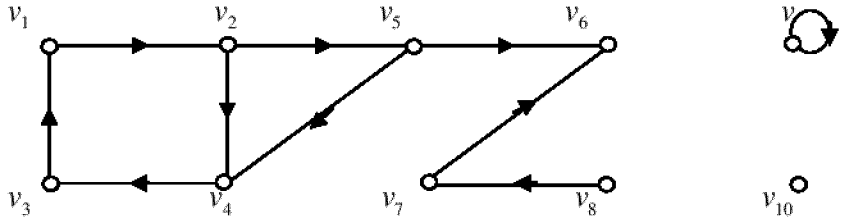


Graph-1

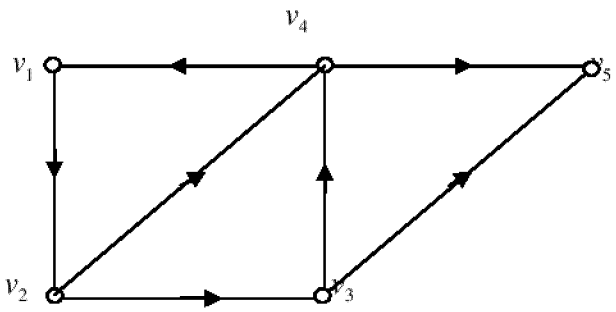


Graph-2

(2) Define a node base. Is the set $\{v_5, v_8, v_9, v_{10}\}$ a node base for the following digraph? Justify your answer. **03**

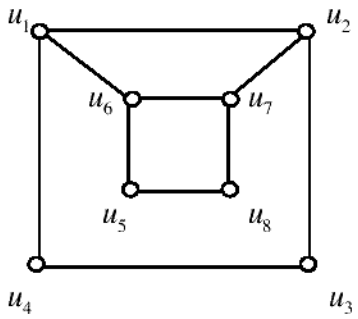


(b) Define (i) a weakly connected graph (ii) a unilaterally connected graph (iii) a strongly connected graph. **07**
Find (i) the weak components (ii) the unilateral components (iii) the strong components of the following digraph.

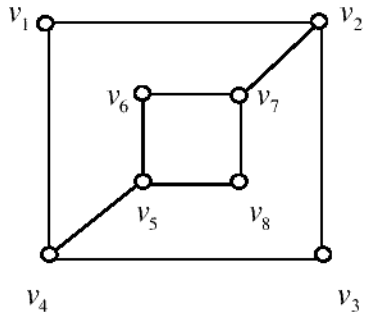


OR

Q.5 **(a)** (1) Define isomorphic graphs. State whether the following digraphs are isomorphic or not. Justify your answer. **04**

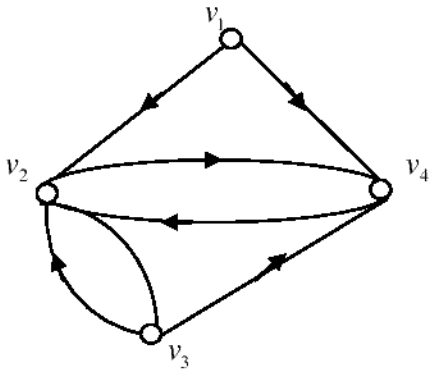


Graph-1



Graph-2

(2) Obtain the adjacency matrix of the following digraph. **03**



Also write the elementary paths of lengths 1 and 2 from v_1 to v_4

(b) Define (i) a binary tree (ii) a complete binary tree. **07**
Show that in a complete binary tree the total number of edges is given by $2(n_t - 1)$, where n_t is the number of terminal nodes.

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