

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER-I • EXAMINATION – SUMMER • 2014

Subject Code: 610003

Date: 18-06-2014

Subject Name: Discrete Mathematics for Computer Science

Time: 10:30 am - 01:00 pm

Total Marks: 70

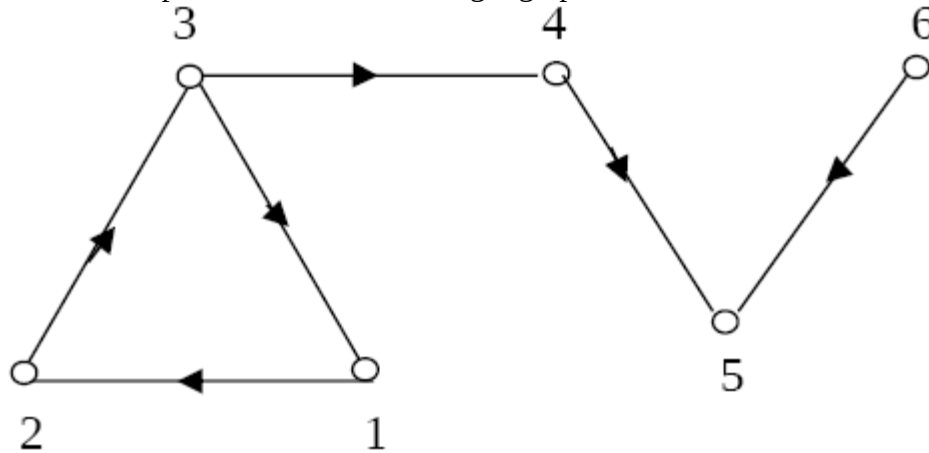
Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** Define the following terms with proper example: [2+2+2+1]
1. Universal Quantifier
2. Existential Quantifier
3. Tautology
4. Contradiction
- (b)** 1. Define Join Irreducible elements and Atoms with proper example. 04
2. Define Meet Irreducible elements and Anti-atoms with proper example. 03
- Q.2 (a)** 1. Define Group. Check Whether $\langle I, \times \rangle$ forms a group or not where I is the set of Integers and $\times$ is multiplication operation. 03
2. Define Sum-Of Product canonical form. Write Boolean Expression $x_1 * x_2$ in an equivalent sum-of-product canonical form in three variables x_1, x_2 and x_3 . 04
- (b)** Define left coset and right coset. Find the left cosets of $\{[0], [3]\}$ in the group $\langle \mathbb{Z}_6, +_6 \rangle$. State Lagrange's Theorem. [2+3+2]
- OR**
- (b)** Define equivalence relation. 07
Let Z be the set of integers and R be the relation called Congruence modulo 5" defined by
 $R = \{ \langle x, y \rangle \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z} \wedge (x - y) \text{ is divisible by } 5 \}$
Show that R is an equivalence relation. Determine the equivalence classes generated by the elements of Z.
- Q.3 (a)** Define Cyclic group. Prove that $\langle \mathbb{Z}_4, +_4 \rangle$ is isomorphic to $\langle \mathbb{Z}_5, *_5 \rangle$ where $\mathbb{Z}_5^* = \mathbb{Z}_5 - \{0\}$. [1+6]
- (b)** Find a minimal sum-of-product form using K-map 07
(i) $\alpha(x, y, z) = xyz + xyz' + x'yz' + x'y'z$
(ii) $\alpha(x, y, z) = xyz + xyz' + xy'z + x'yz + x'y'z$
- OR**
- Q.3 (a)** Define Sub Boolean Algebra. Find all sub-algebras of Boolean algebra $\langle S_{30}, *, \oplus, ', 0, 1 \rangle$. Write proper steps. 07
- (b)** (i) Show that in a lattice if $x \leq y \leq z$ then $x \oplus y = y * z$ 03
(ii) In a lattice show that $(a * b) \oplus (c * d) \leq (a \oplus c) * (b \oplus d)$. 04

Q.4 (a)1. Define weakly connected, unilaterally connected and strongly connected graphs. **03**

2. Define weak, unilateral and strong components. Find the strong, unilateral and weak components for the following digraph. **04**



(b) Draw di-graph and find in-degree and out-degree of each vertex from the given adjacency matrix. Using adjacency matrix, find total numbers of path of length 1 and 2 between each vertex. **07**

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

OR

Q.4 (a) Define Sub-group. Find the subgroups of $\langle \mathbb{Z}_{10}, +_{10} \rangle$ **07**

(b) Define “Lattice as an Algebraic System”, “Complete Lattice” and “Distributive Lattice”. **[3+4]**

Let the sets $S_0, S_1, \dots, S_7$ be given by

$S_0 = \{a, b, c, d, e, f\}$, $S_1 = \{a, b, c, d, e\}$, $S_2 = \{a, b, c, e, f\}$, $S_3 = \{a, b, c, e\}$, $S_4 = \{a, b, c\}$, $S_5 = \{a, b\}$, $S_6 = \{a, c\}$, $S_7 = \{a\}$

Draw the diagram of $\langle L, \subseteq \rangle$,

where $L = \{S_0, S_1, S_2, \dots, S_7\}$

Q.5 (a) Describe the application of Boolean algebra to Relational Database with example. **07**

(b) Find minimal SOP using Quine Mc-cluskey method **07**
 $F(a,b,c,d) = \Sigma (0,1,2,5,6,7,8,9,10,14)$

OR

Q.5 (a) 1. Define Compatibility Relation, Partial order relation with proper example. **03**

2. Let R and S be two relations on a set of positive integers I , **02**

$R = \{ \langle x, 3x \rangle / x \in I \}$

$S = \{ \langle x, 4x \rangle / x \in I \}$

Then find $R \circ R$ and $R \circ S \circ R$.

3. Let $S = \{2,4,5,10,15,20\}$ and the relation $\leq$ is the divisibility relation. **02**

Draw the Hasse diagram of $\langle S, \leq \rangle$.

(b) Show that the lattice $\langle \mathbb{Z}_n, D \rangle$ for $n=100$ is isomorphic to the direct product of lattice for $n=4$ and $n=25$. **07**
