

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER-I • EXAMINATION – WINTER • 2015

Subject Code: 610003

Date: 30-12-2015

Subject Name: Discrete Mathematics for Computer Science

Time: 10:30 am - 01:00 pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Define Boolean Expression and show that $[a * (b' + c)'] * [b' + (a * c')'] = a * b * c'$ **07**
- (b) Use the Quine McClusky method to simplify the SOP expansion **07**
1. $F(a,b,c,d) = \sum(0,1,4,5,9,11)$
 2. $F(a,b,c,d) = \sum(10,12,13,14,15)$
- And draw the circuit diagram of minimized function.
- Q.2** (a) Answer the following. **07**
1. Prove that if “All men are mortal” and “Socrates is a man” then “Socrates is a mortal”. By using theory of Inference.
 2. By using contradiction method prove that $\sqrt{2}$ is an irrational number.
- (b) Answer the following. **07**
1. Define isomorphic lattices. Draw the Hasse diagrams of lattices
(i) $(S_4 \times S_{25}, D)$
(ii) (S_{100}, D)
Check whether these lattices are isomorphic?
 2. Show that the operation of meet & join on a lattice are commutative, associative and idempotent.
- OR**
- (b) Answer the following. **07**
1. Without constructing the truth table prove that $\sim p \rightarrow (p \rightarrow q)$ is tautology.
 2. Define Sub lattice. Find all the sub lattices of $\langle S_{30}, D \rangle$.
- Q.3** (a) Answer the following. **07**
1. By using the truth table prove that $\sim p \rightarrow (p \rightarrow q)$ is tautology.
 2. Find all the sub lattices of $\langle S_{10}, D \rangle$.
- (b) Define Normal Subgroup. Let $\langle Z_6, +_6 \rangle$ be the group and $H = \{0, 3\}$ be the subgroup of Z_6 . Verify that H is normal subgroup of Z_6 . **07**
- OR**
- Q.3** (a) If $A = \{1, 2, 3, 4\}$ & $R = \{ \langle x, y \rangle / x = y \}$. Verify Reflexive, Symmetric, Transitive, Irreflexive for given set of relation R . **07**
- (b) Define “composite relation” & “converse of a relation”. Find the relation matrix M_R of a relation $R = \{ \langle a, a \rangle, \langle a, c \rangle, \langle b, a \rangle, \langle b, b \rangle, \langle c, a \rangle, \langle c, b \rangle, \langle c, c \rangle \}$ on the set $\{a, b, c\}$. Find the relation matrices of $\sim M_R$ & $M_{R \circ \sim M_R}$. **07**
- Q.4** (a) Define Group, Sub group, Left Coset and also prove that Inverse of the group $\langle G, * \rangle$ is unique. **07**

(b) Prove the following. 07

1. A subset $S \neq \Phi$ of G is a subgroup of $\langle G, * \rangle$ iff for any elements $a, b \in S$, $a*b^{-1} \in S$.
2. If group $\langle G, * \rangle$ is abelian group then show that $(a*b)^n = a^n*b^n$.

OR

Q.4 (a) Prove that $\langle Z_5^*, X_5 \rangle$ and $\langle Z_4, +_4 \rangle$ are isomorphic where $Z_5^* = Z_5 - \{0\}$. 07

(b) Define Cyclic group. Is $\langle Z_5, +_5 \rangle$ a cyclic group? If Yes, find its generators. 07

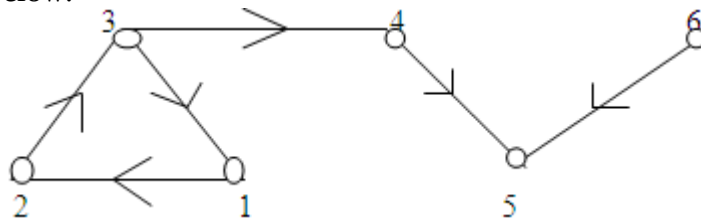
Q.5 (a) Give definition of graph. When two graphs are said to be isomorphic? Give at least two examples of graphs which are isomorphic. 07

(b) Give three other representation of tree expressed by $(V_0(V_1(V_2)(V_3)(V_4))(V_5(V_6)(V_7)(V_8)(V_9))(V_{10}(V_{11})(V_{12})))$ And also obtain binary tree corresponding to it. 07

OR

Q.5 (a) Define Strong component and Unilateral component. 07

Find the Strong components and Unilateral components of the diagram given below.



(b) Define Complete binary tree. Show through two examples with $n_t=7$ and $n_t=8$ of complete binary trees that the total number of edges is given by $2(n_t - 1)$, where n_t is the number of terminal nodes. 07
