Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY
MCA SEMESTER– I• EXAMINATION – WINTER 2016

Subject Code:610003 **Date:03/01/2017**
Subject Name: Discrete Mathematics for Computer Science
Time:10.30 AM TO 01.00 **Total Marks: 70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** i) Consider the statement, "If today is Monday, then I will go for a walk". Write converse, inverse and contrapositive for the given statement. **03**

ii) What is Discrete Mathematics? State the importance of it. **04**

(b) Using indirect proof technique, show that if a^2+3 is odd, then a is even. **07**

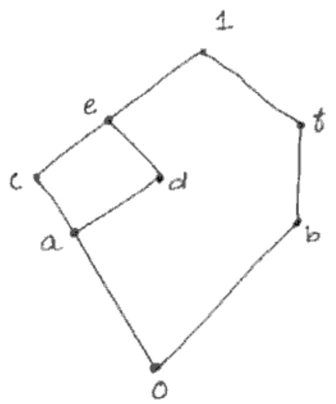
Q.2 **(a)** Let X = {1,2,3,4} and R = {(x, y) / x > y} be relation on it.
(i) Write properties of R. **03**
(ii) Write matrix of R. **02**
(iii) Draw graph of R. **02**

(b) i) Define: Equivalence Relation. Let R = {(1,2), (3,4), (2,2)} and S = {(4,2), (2,5), (3,1), (1,3)}. Find R ° S, S ° R, and R ° R. **04**

ii) Test the validity of the logical consequences: **03**
All dogs fetch.
Kitty does not fetch.
Therefore Kitty is not a dog.

OR

(b) Define: Lattice. Determine join-irreducible elements, meet-irreducible elements, atoms and anti-atoms for the lattices shown in the Figure given below: **07**



Q.3 **(a)** Use the Quine McClusky method to simplify the SOP expansion, **07**
$f(a, b, c, d) = \Sigma\,(0, 2, 4, 6, 8, 10, 12, 14)$

(b) Define: Isomorphic Lattices. Draw the Hasse diagrams of lattices **07**

i) $(S_4 \times S_{25}, D)$

ii) (S_{100}, D)

Check whether these lattices are isomorphic?

OR

Q.3 **(a)** Use K - map representation to find a minimal sum-of-product expression **07**

i) $f(a, b, c, d) = \sum (10, 12, 13, 14, 15)$

ii) $f(a, b, c, d) = \sum (0, 1, 2, 3, 13, 15)$

(b) i) Define a sub-lattice. Give any four sub-lattices of the lattice (S_{12}, D). **03**

ii) In poset (S_{36}, D), find i) GLB X, LUB X (ii) GLB Y, LUB Y where **04**
$X = \{ 4, 6, 12 \}$ and $Y = \{ 3, 6, 9 \}$.

Q.4 **(a)** Define cyclic group. Show that cyclic group is abelian but converse is not true. **07**
Is $<z5, +5>$ a cyclic group? If so, find its generators.

(b) Define subgroup of a group, left coset of a subgroup $<H, *>$ in the group $<G,*>$. **07**
Find left cosets of $\{[0], [3]\}$ in the group $<Z6, +6>$.

OR

Q.4 **(a)** i) Show that in a group $<G, *>$, if for any a, b $\in$ G, $(a * b)^2 = a^2 * b^2$, then **03**
$<G, *>$ must be abelian.

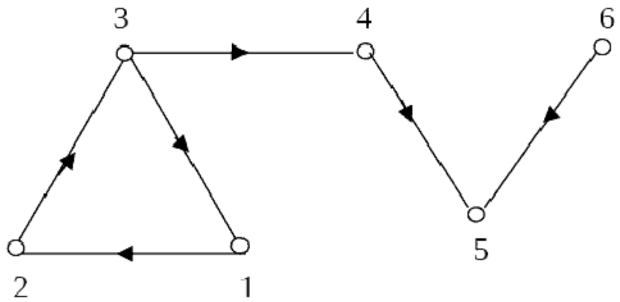
iii) Show that $< \{1,4,13,16\}, *_{17} >$ is a subgroup of $< Z_{17}^*, *_{17} >$. **04**

(b) i) Define: POSET. Let P(x, y) denote the sentence: 2x + y = 1.What are the **04**
truth values of $\forall x \exists y P(x, y)$, $\forall x \forall y P(x, y)$ and $\exists x \exists y P(x,y)$ where the domain of x, y is the set of all integers?

ii) Show without constructing the truth table that the statement formula **03**
$\sim p \rightarrow (p \rightarrow q)$ is a tautology

Q.5 **(a)** Define: Path, Elementary Path, Cycle, Binary Tree, Sling, Isolated Node, **07**
Null Graph

(b) Define weakly connected, unilaterally connected and strongly connected **07**
graphs. Find the strong, unilateral and weak components for the following digraph.



OR

Q.5 **(a)** Draw di-graph and find in-degree and out-degree of each vertex from the given adjacency matrix. Using adjacency matrix, find total numbers of path of length 1 between each vertex. **07**

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

(b) Define: Leaf. Draw the graph of the tree represented by (A(B(C(D)(E)))(F(G)(H)(J))(K(L)(M)(N(P)(Q(R))))). Obtain the binary tree corresponding to it. **07**

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