Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER–I• EXAMINATION – WINTER 2017

Subject Code: 610003 Date: 01-01-2018

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 10:30 am to 01:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define: Boolean algebra. Draw Hasse diagram of (S_{30}, D) and determine all the sub Boolean of the Boolean algebra ‹ S_{30}, Λ, V, ‘ , 0 , 1 ›. **07**

(b) Use Predicate, Quantifier and logical connectives, Rules of inferences, Argument forms to prove the following consequence is valid. **07**
All athletes are healthy. All healthy people take vitamins. Grant is an athlete. Therefore Grant takes vitamin.

Q.2 **(a)** Define Lower bound and greatest lower bound. Let **07**
P = < {3, 5, 9, 15, 24, 45},D> be a poset. Draw the Hasse diagram. Find
i)The maximal element & minimal element.
ii) The greatest and the least element.
iii)The lower bounds of {3, 5}, if any & the upper bound of { 9, 15 } , if any.
iv) GLB of {15, 45} & LUB of {3, 9, 15}.

(b) Using proof by contradiction method, prove that $\sqrt{2}$ is an irrational number. **07**

OR

(b) Define an equivalence relation. Prove that the relation “congruence modulo 3” given by ≡ {<x, y> / x-y is divisible by 3} over the positive integer is an equivalence relation. **07**

Q.3 **(a)** Define: Sub lattice. Verify whether the given sets are sub lattice of (S_{30},D) or not. (i) [{1,2,3,6} ,GCD , LCM] (ii) [{1,3,6, 15 } ,GCD , LCM] **07**

(b) Use the Karnaugh map representation to find a minimal sum-of-products expression of f(a ,b , c , d) = ∑ (0,1, 2,3,13,15). **07**

OR

Q.3 **(a)** Define complemented lattice. Which of two lattices <Sn, D> for n=30 and n=45 are complemented? Draw Hasse Diagram of these lattices. **07**

(b) Use the Quine McClusky method to simplify the SOP expansion, **07**
f(a ,b , c , d) = ∑ (0, 2,4,6,8, 10, 12, 14).

Q.4 **(a)** Define: Group and Abelian group. Show that in a group (G, *), **07**
if for every a, b € G, $(a * b)^2 = a^2 * b^2$, then (G, *) is an Abelian group.

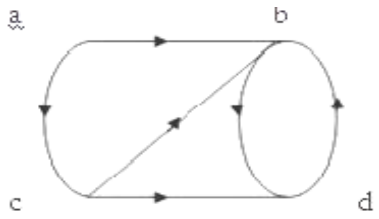
(b) Define :Left coset of a subgroup <H, *> in the group<G, *>. **07**
Find left cosets of {[0], [3]} in the group <Z_6, +6>.

OR

Q.4 **(a)** Define Cyclic group. Show that $(Z^*_5, *_5)$ is a cyclic group and find all the generators of $(Z^*_5, *_5)$. Where $Z^*_5 = Z_5$-[0] **07**

(b) Show that the symmetric set (S_3, o) is a group. Is it abelian? Justify. **07**

Q.5 **(a)** Define Nodebase. Find Nodebase of the following digraph. **07**



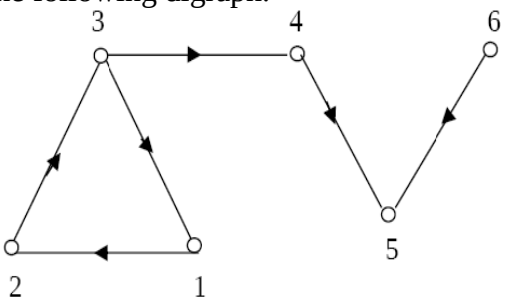
(b) Define adjacency matrix. Draw di-graph of given adjacency matrix.Find in-degree & out-degree of each vertex from the given adjacency matrix. **07**

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

OR

Q.5 **(a)** Define: Strong component, Unilateral component and weak component of the digraph. **07**

Determine Strong component, Unilateral component and weak component of the following digraph.



(b) Define tree. Give three different representation of the given tree. **07**

(v0(v1(v2)(v3)(v4))(v5(v6)(v7)(v8)(v9))(v10(v11)(v12)))

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