

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY
MCA Integrated - SEMESTER-II • EXAMINATION – SUMMER • 2014

Subject Code: 4420601

Date: 18-06-2014

Subject Name: Discrete Mathematics for Computer Science

Time: 10:30 am - 01:00 pm

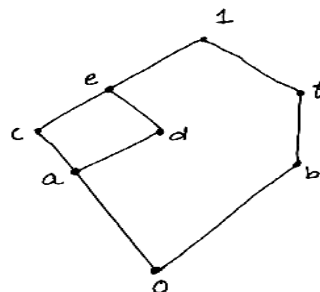
Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** With proper justification give an example of **07**
- i) A bounded lattice which is complemented but not distributive.
 - ii) A bounded lattice which is distributive but not complemented.
 - iii) A bounded lattice which is neither distributive nor complemented.
 - iv) A bounded lattice which is both distributive and complemented.
- (b)** When a poset is said to be a lattice? Explain. Is every poset a lattice? Justify. **07**
Is the poset $\{\Phi, \{p\}, \{q\}, \{p,q,r\}, \subseteq\}$ a lattice?

- Q.2 (a)** Define: Chain. Determine join-irreducible elements, meet-irreducible elements, atoms and anti-atoms for the lattices shown in the Figure below: **07**



- (b)** Define: Boolean Algebra. Find all Sub Boolean algebra of Boolean algebra $\langle S_{30}, \wedge, \vee, ', 0, 1 \rangle$. **07**

OR

- (b)** i) Given an expression $\alpha(a,b,c,d) = \sum (2,3,6,8,12,15)$, determine the value of $\alpha(3,5,10,30)$ where $3,5,10,30 \in \langle S_{30}, D \rangle$ **05**
- ii) Show that **02**
- a) $a + a' = 1$
 - b) $a + 0 = a$
- where $a + b = (a * b') \text{ join } (a' * b)$

- Q.3 (a)** Use the K-map representation to find a minimal sum-of-products expression for the following function: **07**
- a) $f(a,b,c,d) = \sum (0,5,7,8,12,14)$
 - b) $f(a,b,c,d) = \sum (5,7,10,13,15)$
- (b)** Use the Quine-McCluskey representation to find a minimal SOP expression: **07**
 $f(a,b,c,d) = \sum (0,1,6,7,8,9,13,14,15)$

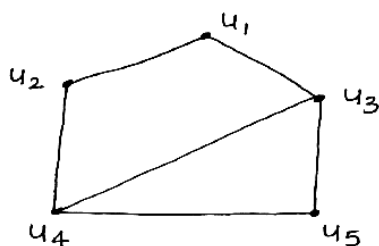
OR

- Q.3 (a)** i) Let $(B, *, ', 0, 1)$ be a boolean algebra in any Boolean algebra prove the following: $a = b \Leftrightarrow ab' + a'b = 0$. **03**
- ii) Find the sum of products expression of Boolean function. **03**
 $f(x,y,z) = (x + z) * y$
- iii) Define: Sub-Lattice. **01**

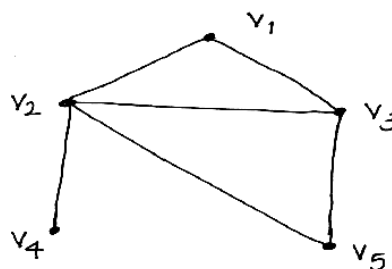
- (b) Show that the lattice $\langle S_n, D \rangle$ for $n = 216$ is isomorphic to the direct product of lattices for $n = 8$ and $n = 27$. 07
- Q.4** (a) i) Define an abelian group. Show that if every element in a group is its own inverse, then the group must be abelian. 03
 ii) Show that every subgroup of a cyclic group is normal. 03
 iii) Let $(G, *)$ be a group. Let $G = 5$. How many subgroups are there of G ? Why? 01
- (b) Show that the set of all positive rational number forms an abelian group under the composition defined by $a * b = ab/2$. 07
- OR**
- Q.4** (a) Define: Group & Cyclic Group. Find the generator of $(Z_5^*, *)$. 07
 (b) Find in-degree and out-degree of each node from the following adjacency matrix A and draw its diagram. 07

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

- Q.5** (a) Define: Binary tree, Loop, Null graph. Show through two examples with $n_t = 7$ and $n_t = 8$ of complete binary trees that the total number of edges is given by $2(n_t - 1)$, where n_t is the number of terminal nodes. 07
- (b) Define Directed tree. Give three different tree representations of the following: 07
 $(v_0(v_1(v_2)(v_3)(v_4))(v_5(v_6)(v_7)(v_8)(v_9))(v_{10}(v_{11})(v_{12})))$
- OR**
- Q.5** (a) Define: Isomorphic Graph, Edge Simple. Verify that, are the following graphs are isomorphic? 07



Graph - 1



Graph - 2

- (b) Define: Node Base of a simple digraph. Find the reachability set of all nodes for the digraph given in figure given below: Also find the nodebase for it. Is the graph Strongly or unilaterally connected?

07

