Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA Integrated - SEMESTER–II • EXAMINATION – SUMMER • 2015

Subject Code: 4420601 Date: 28-05-2015

Subject Name: Discrete Mathematics for Computer Science

Time: 10:30 am - 01:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define poset. When is a poset said to be a lattice? Draw Hasse diagrams of following posets and examine which of them are lattices. **07**

(a) < P(S), $\subseteq$ >, S = {a, b, c}
(b) <{1, 2, 3, 12, 18}, D >
(c) <{1, 2, 3, 6}, D >
(d) < S_{16}, D >.

(b) Define: Boolean Algebra. Find all Sub Boolean Algebra of Boolean Algebra <S_{30}, ş , V, ǵ0,1 >. **07**

Q.2 **(a)** (1) Draw Hasse- diagram for the following : **04**

i) < S_{105},D >
ii) < S_{70},D >

(2) With proper justification give an example of **03**

a) A bounded lattice which is complemented but not distributive.
b) A bounded lattice which is distributive but not complemented.
c) A bounded lattice which is both distributive and complemented.

(b) Define: **07**

i) Join irreducible elements.
ii) Atoms of a Boolean algebra.

Determine Join-irreducible elements and atoms of following Boolean algebra also draw the Hasse Diagram:

i) (S_{210}, D)
ii) ⟨ P(S), ž ,∪, ', ū , S⟩ where S = {a, b, c}

OR

(b) Define Lower bound and Upper bound. Let P = < 3, 5, 9, 15, 24, 45}, D> be a poset. Draw the Hasse diagram. Find **07**

i) maximal element. & minimal element.
ii) the greatest and least element.
iii) the lower bounds of {3, 5}, if any & the upper bound of { 9, 15 } , if any
iv) GLB of {15, 45} & LUB of {3, 9, 15}.

Q.3 **(a)** Show that the lattice < S_n, D > for n=100 is Isomorphic to the direct product of lattice for n=4 & n=25. **07**

(b) Use the Quine-McCluskey algorithm to find the prime implicants of the expression: f(a, b, c, d) = × (0, 1, 4, 5, 9, 11). Also obtain a minimal expression for the same. **07**

OR

Q.3 **(a)** Use K-map to find a minimal SOP expression for the function given by × (0,1,2,3,6,7,13,14) in four variables w,x,y,z. **07**

(b) (1) Explain Stoneôs representation theorem by giving a suitable example. **03**
(2) Obtain the sum-of-products canonical form of the following Boolean expressions: **04**
i) (x1 ⊕ x2)N⊕ (x1Nj x3)
ii) (x1 * x2N⊕ x3

Q.4 **(a)** (1) Define Sub-lattice. Write any four sublattices of (S_{12}, D). **03**
(2) Describe the application of Boolean algebra to Relational Database. **04**

(b) (1) Show that in a group < G, * > if for any a , b ∈ G, $(a * b)^2 = a^2 * b^2$, then < G, * > must be abelian. **03**
(2) Show that $< \{1,4,13,16\}, x_{17} >$ is a subgroup of $< Z_{17}^*, x_{17} >$. **04**

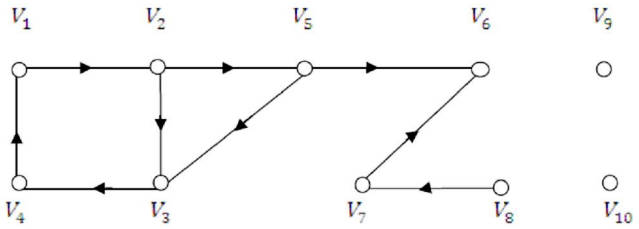
OR

Q.4 **(a)** Show that the set of all positive rational number forms an abelian group under the composition defined by a * b = ab/2 . **07**

(b) Define Cyclic Group. Prove that $< Z_7^*, x_7 >$ is a group. Also find generators of this group. **07**

Q.5 **(a)** Define : Forest, Binary Tree, Node Base, Cycle, Elementary Path, Isolated Node, Graph. **07**

(b) Define node base of a simple digraph. Find reachablility set of all nodes for the following diagraph. **07**



OR

Q.5 **(a)** Define: Isomorphic Graph, Sling and Weighted Graph. State weather the following digraphs are isomorphic or not: **07**

(b) Define: Directed tree and its leaf. Draw the graph of the tree represented by (A(B(C(D)(E)))(F(G)(H)(J))(K(L)(M)(N(P)(Q(R))))). Obtain the binary tree corresponding to it. **07**

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